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Dissertation

**PRODUCT SEQUENCING AND BLENDING OF RAW MATERIALS TO FEED ARC
FURNACES: A DECISION SUPPORT SYSTEM FOR A MINING-METALLURGICAL
INDUSTRY**

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**Ouro Preto
Minas Gerais, Brazil
2021**

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INDUSTRY**

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on Instrumentation, Control and Automation of
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*To my parents, Walter de Paula
Bacharel and Marta da
Consolação Freitas Bacharel, for
always believing in me, and for all
the effort and difficulties they went
through to educate me. To my
brother Fernando de Freitas
Bacharel and my wife, Kamila
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being always present, supporting
me and above all encouraging me
with love.*

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I thank God for being present at all times, always giving me strength and protection during the walk. I thank God for always making me realize that goals like this are perfectly possible for those who believe in Him and are dedicated to achieving them.

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Abstract

Abstract of Dissertation presented to the Graduate Program on Instrumentation, Control and Automation of Mining Process as a partial fulfillment of the requirements for the degree of Master of Science (M.Sc.)

PRODUCT SEQUENCING AND BLENDING OF RAW MATERIALS TO FEED ARC FURNACES: A DECISION SUPPORT SYSTEM FOR A MINING-METALLURGICAL INDUSTRY

Rafael de Freitas Bacharel

September/2021

Advisors: Marcone Jamilson Freitas Souza

Luciano Perdigão Cota

A large amount of data available today and the complex situations present in the industry make decision support systems increasingly necessary. This work deals with a problem of a mining-metallurgical industry in which the production of products used to feed arc furnaces must be sequenced in work shifts. There is a due date and a quality specification for each product. These products are generated from raw materials available in a set of silos and must satisfy the required quality specifications. The aim is to minimize the total production time and the total tardiness. To solve it, we developed a decision support system that applies a matheuristic algorithm to do the product schedule and determine the amount of raw material to produce each product. In the proposed algorithm, the products generated in each work shift are chosen through a dispatch heuristic rule based on the shortest production time. In turn, the amount of raw material to be used is calculated by solving a goal linear programming formulation of a blending problem. We generate instances that simulate real cases to evaluate the developed algorithm. The results generated for these instances show a good performance of the proposed algorithm, validating its use as a tool to support decision-making.

Keywords: Arc furnace, Decision support system, Blending, Matheuristic, Operations research in industry.

Macrotheme: Power Plant; **Research Line:** Information, Communication and Industrial Automation Technologies; **Theme:** Product quality improvement;

Resumo

Resumo da Dissertação apresentada ao Programa de Pós Graduação em Instrumentação, Controle e Automação de Processos de Mineração como parte dos requisitos necessários para a obtenção do grau de Mestre em Ciências (M.Sc.)

SEQUENCIAMENTO DA PRODUÇÃO E BLENDAGEM DE MATÉRIAS-PRIMAS PARA ALIMENTAÇÃO DE FORNOS A ARCO: UM SISTEMA DE SUPORTE À DECISÃO PARA UMA INDÚSTRIA MINERO-METALÚRGICA

Rafael de Freitas Bacharel

September/2021

Orientadores: Marcone Jamilson Freitas Souza

Luciano Perdigão Cota

A grande quantidade de dados disponíveis hoje e as situações complexas presentes na indústria tornam os sistemas de apoio à decisão cada vez mais necessários. Este trabalho trata de um problema de uma indústria minero-metalúrgica na qual a produção dos produtos utilizados na alimentação dos fornos a arco deve ser sequenciada em turnos de trabalho. Para cada produto há uma data de vencimento e uma especificação de qualidade. Esses produtos são gerados a partir de matérias-primas disponíveis em um conjunto de silos e devem atender às especificações de qualidade exigidas. O objetivo é minimizar o tempo total de produção e o atraso total. Para resolvê-lo, desenvolvemos um sistema de apoio à decisão que aplica um algoritmo matheurístico para sequenciar a produção e determinar a quantidade de matéria-prima necessária para produzir cada produto. No algoritmo proposto, os produtos gerados em cada turno de trabalho são escolhidos por meio de uma regra heurística de despacho baseada no menor tempo de produção. Por sua vez, a quantidade de matéria-prima a ser utilizada é calculada resolvendo uma formulação de programação linear por metas para o problema de mistura. Geramos instâncias que simulam casos reais para avaliar o algoritmo desenvolvido. Os resultados gerados para estas instâncias mostram um bom desempenho do algoritmo proposto, validando sua utilização como ferramenta de apoio à tomada de decisão.

Palavras-chave: Forno a arco, Sistema de suporte à decisão, Blendagem, Matheurística, Pesquisa operacional na indústria.

Macrotema: Usina; **Linha de Pesquisa:** Tecnologias da Informação, Comunicação e Automação Industrial; **Tema:** Melhoria da qualidade de produtos;

List of Figures

Figure 1.1	Typical flowchart of unit operations in the mineral industry. Adapted from Luz <i>et al.</i> (2010).	13
Figure 1.2	Typical flowchart of production stages in the metallurgical industry. Adapted from Seetharaman <i>et al.</i> (2014).	14
Figure 1.3	General flowchart of the approached process.	15
Figure 2.1	Scheduling example.	19
Figure 4.1	Example of setting up silos for storing blends and tripper car.	27
Figure 4.2	Example of setting up silos for storing blends.	28
Figure 7.1	User interface for the decision support system.	39
Figure 7.2	Behavior of raw material silos for instance 4.	40
Figure 7.3	Behavior of raw material silos for instance 5.	40
Figure 7.4	Behavior of raw material silos for instance 6.	41
Figure 7.5	Behavior of raw material silos for instance 7.	41
Figure 7.6	Schedule generated for the instance with 30 products.	42

List of Tables

Table 2.1	Job processing times on identical parallel machines.	19
Table 2.2	Processing time of jobs on unrelated parallel machines.	20
Table 4.1	Production planning to be executed.	28
Table 4.2	Characteristics of raw material silos.	29
Table 7.1	Characteristics of the instances.	38
Table 7.2	Weights for the quality parameters.	38
Table 7.3	Results of the proposed algorithm per instance.	39

List of Acronyms and Abbreviations

ARPD Average Relative Percentage Deviation

ASPT Adaptive Shortest Processing Time

DLL Dynamic-link library

DSS_ASPT Algorithm to decision support

EDD Earliest Due Date

GA Genetic Algorithm

GRASP Greedy Randomized Adaptive Search Procedure

LFT Least Flexible Job First

LS-SVM Least Squares Support Vector Machine

MILP Mixed-Integer Linear Programming

MOPSO Multi-objective Particle Swarm Optimization

MOVNS Multi-objective General Variable Neighborhood Search

NSGA-II Non-dominated Sorting Genetic Algorithm II

PSO Particle Swarm Optimization

RMF Raw Material Feed

RPT Rejected Products Treatment

SA Simulated Annealing

SPT Shortest Processing Time

SQP Sequential quadratic programming

SVR Support Vector Regression

XGBoost Extreme Gradient Boosting Algorithm

List of Symbols

α Parameter that reflects the importance of each parcel of the objective function

a Index for raw material silo

b Index for quality parameter

c_a Cost of one tonne of material a

C_j Completion time of the product j

C_{\max} Makespan of a solution

CB Set of conveyor belts

d_j Due date of product j

dnm_b Negative deviation from the grade target for the parameter b

$dnSiloSM_a$ Amount of raw material remaining in the silo a used for blending

dpm_b Positive deviation from the grade target for the parameter b

fo Objective function value

h Instant of the planning horizon

H Planning horizon

i Index for machine

j Index for product

M Set of materials

p_j Processing time of product j

PR Set of products

Q_a Quantity available for material a

Q_{des} Total amount desired for the blending

Ql Minimum storage value of the silo

Qu Maximum storage value of the silo

S Set of quality parameters for raw materials and products

s Solution

SF Set of screw feeders

SM Set of silos to store the raw material

T Total tardiness of a solution

T_j Tardiness of the product j

t_{ab} Grade of the parameter b of material a

te_{jb} Grade encountered for the parameter b in the product j

tl_b Lower bound for the parameter b in the blending

tl_{jb} Lower bound for the control parameter b in product j

tr_b Target for the parameter b in the blending

tr_{jb} Target for the parameter b in the product j

tu_b Upper bound for the parameter b in the blending

tu_{jb} Upper bound for the control parameter b in product j

w_{mb} Weight for the deviations from the grade target of the parameter b

ws_a Weight for the use of the silo a

x_a Quantity to be used of the material a in the blend

$type_a$ Binary parameter that assumes value 1 if the raw material silo a has the same type of material desired for the product j to be blended and 0 otherwise

Summary

1	Introduction	13
1.1	Motivation	16
1.2	Objectives	17
1.2.1	General objective	17
1.2.2	Specific objectives	17
1.3	Methodology	17
1.4	Work structure	18
2	Theoretical Background	19
2.1	Scheduling problem	19
2.2	Blending problem	20
3	Literature Review	23
3.1	Arc furnaces	23
3.2	Scheduling in mining and metallurgical industries	23
3.3	Blending in mining and metallurgical industries	24
4	Problem Statement	26
5	The Mathematical Formulation of the Blending Problem	30
6	Proposed Algorithm	32
6.1	Complementary functions	34
6.1.1	RPT function	34
6.1.2	RMF function	35
7	Computational Experiments	37
8	Conclusions and Future Works	43
9	Publications	44
	References	45

1. Introduction

The mining and metallurgy industries are essential for the country's economic development. Mining is responsible for more than 20% of Brazilian exports in the last ten years, and metallurgy is the primary destination for foreign direct investment in the industry, according to Carvalho *et al.* (2017). These strategic sectors continuously seek to incorporate knowledge and technology to increase production and profit, reduce costs, and reduce environmental impacts.

According to Luz *et al.* (2010), the processing of ores consists of operations that alter the granulometry, concentration, or shape of minerals. A rock consisting of a mineral or aggregate of minerals containing one or more valuable minerals is characterized as an ore. The operations carried out in the processing of ores, also known as unitary operations, consist briefly of extracting the ore, comminution, concentration, and disposal of the tailings. The flowchart in Figure 1.1 shows the typical unit operations of the mineral industry.

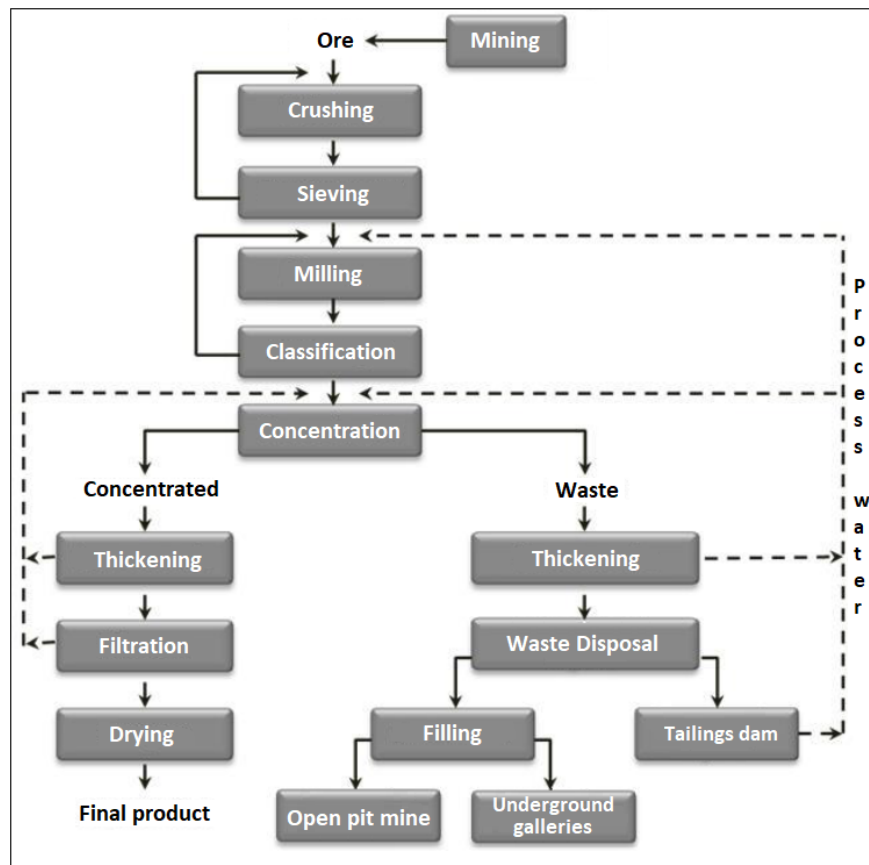


Figure 1.1: Typical flowchart of unit operations in the mineral industry. Adapted from Luz *et al.* (2010).

The metallurgical industry is responsible for transforming ores or impure or raw metals into metals free of impurities, which can therefore be used in final applications such as sheets and tubes, among others. According to Seetharaman *et al.* (2014), its production process typically involves the stages of separation, formation of the metallic solution, production of the

crude metal, purification, and preparation of the product, as shown in Figure 1.2.

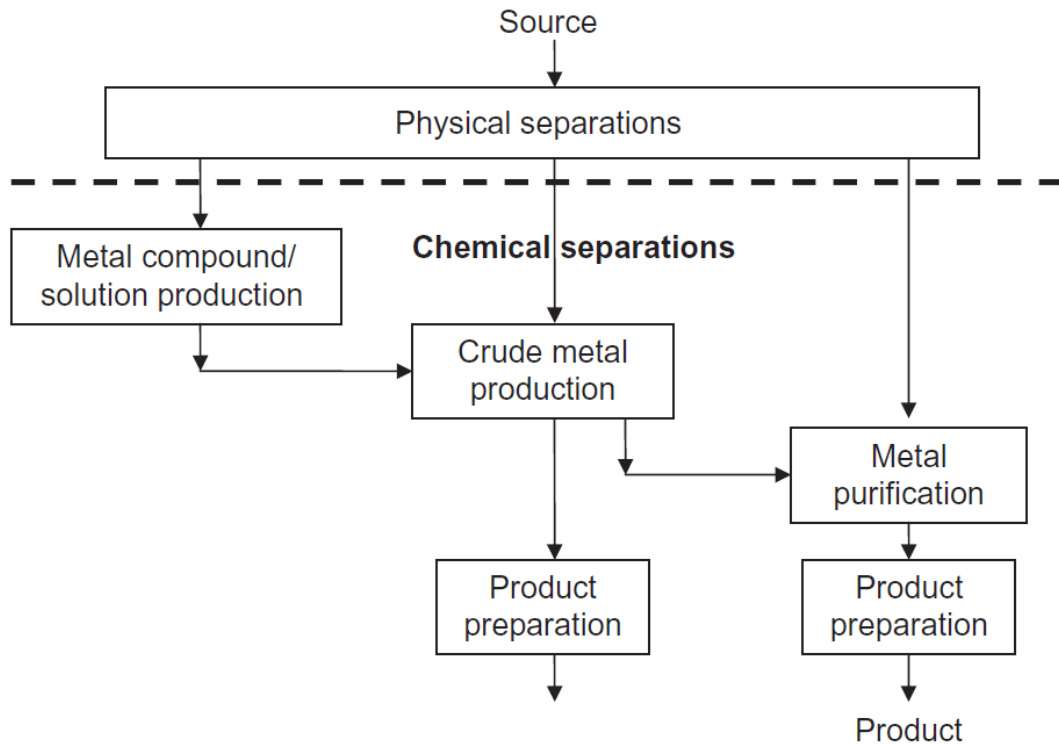


Figure 1.2: Typical flowchart of production stages in the metallurgical industry. Adapted from Seetharaman *et al.* (2014).

Briefly, the following metallurgical processes can be mentioned, according to Seetharaman *et al.* (2014):

- **Pyrometallurgy:** It uses high temperatures for the treatment of high-quality ores and recycled materials. Even with high temperatures, this process has a low energy consumption as it can use exothermic chemical reactions and energy from second-generation systems. Among metallurgical processes, pyrometallurgy represents the cleanest process with the most negligible environmental impact.
- **Hydrometallurgy:** This process uses chemical reactions from aqueous solutions to separate metals from low-quality ores. Due to this, its processing rate is limited to the speed of reactions. The treatment of effluents generated to remove harmful substances must be taken into account to reduce the environmental impact of this type of process.
- **Electrometallurgy:** Electrochemical methods are used to separate metals. It has low processing rates and high storage costs. However, it is used for high-value metals.

In mining-metallurgical industries, the ore explored is concentrated and becomes a product used in metallurgical processes. For this work, a pyrometallurgical process is analyzed. First, a primary arc furnace receives as feed the concentrated mineral already agglutinated in

sinter, besides coal and scrap for burning. After the material melts, the slag is separated, and the material flows through a leakage channel where it is conditioned in tanks. This material is then filtered, dried, and stored in silos in subsequent unit operations, as shown in Figure 1.3. With the material ready for use, it is then removed from the silos to generate a mixture to be used in a secondary arc furnace responsible for producing the desired metal alloys.

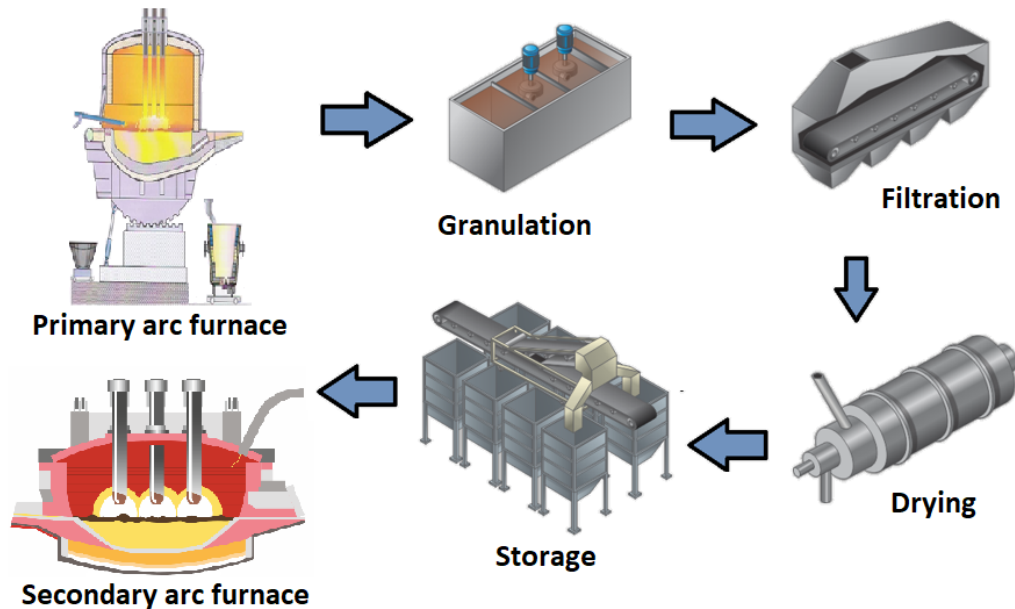


Figure 1.3: General flowchart of the approached process.

Arc furnaces are one of the most important pieces of equipment in the metallurgical industry due to their versatility in producing different types of steel. According to Worldsteel (2020), of all global crude steel production in 2019, 27.9% resulted from arc furnaces. Usually, an arc furnace is fed by scrap and elements from different alloys, allowing the adjustment of the steel's chemical composition to be generated.

Equipment such as arc furnaces still has a low level of automation in its operation, leaving the production decisions to be carried out by its operators according to their experiences Shyamal and Swartz (2017). A survey carried out by Olivier and Craig (2017) on the degree of automation in the global mineral processing industry shows that operators' actions on processes are still frequent. Given the massive amount of data generated by industrial processes and the different situations for decision making, decision support systems are increasingly important, as they allow a complete analysis of the entire production chain Liu and Zaraté (2014).

The present work deals with a problem of a mining-metallurgical industry in which the production that feeds arc furnaces must be sequenced in work shifts. In the company under study, this task is done manually.

Sequencing the production and performing the correct blending of materials is a problem found in different industry types. For example, in the oil industry, Bayu *et al.* (2020) deals with the scheduling of gasoline blending and distribution. Franzoi *et al.* (2019) address

the scheduling of processing units and blending for gasoline production. Pereira *et al.* (2018, 2020) deals with a crude oil scheduling problem in a refinery using a multi-objective evolutionary algorithm. For the mineral industry, Rezakhah *et al.* (2020) consider ore blending in stockpiles from the open-pit mine scheduling. However, to our knowledge, the problem of product sequencing and blending of raw materials to feed arc furnaces has not yet been addressed in the literature. Feeding an arc furnace with the correct blend of raw materials requires important decisions, as this can affect steel specifications. The blending process allows generating a product with the right proportions of raw material, increasing the quality of production and its economic benefits Lingshuang *et al.* (2013). Besides, scheduling efficiently the products that feed the furnace reduces the total production time and generates more products.

This work proposes a decision support system based on a matheuristic algorithm to perform the production scheduling and determine the blends that form each product in different planning horizons. A heuristic dispatch rule chooses the products to be produced in each work shift. The optimal quantities of raw materials to generate each product are determined through a goal linear programming formulation. As the proposed algorithm combines heuristic and exact procedures, it is considered a matheuristic, according to Ribeiro *et al.* (2020). We generate instances that simulate real cases to evaluate the developed algorithm. The results generated show a good performance of the proposed algorithm, validating its use as a tool to support decision-making.

This work differs from the others in the literature for dealing with the problem of product sequencing and blending of raw materials to feed arc furnaces, considering simultaneously: 1) the blending of raw materials, stored in silos, to generate products obeying lower and upper bounds for their control parameters; 2) the blending aiming to minimize deviations from the grade targets for the control parameters of the products generated; 3) the continuous feeding of silos; 4) the sequencing of these products into work shifts within a planning horizon.

1.1. Motivation

Many decisions related to production processes in industries are still reserved for operators in the control room because it is believed that their experience is the best option to the problems encountered. However, these choices can lead to suboptimal solutions, which increase time and costs.

There are several motivations for carrying out this work. Firstly, the importance of solving a real-world problem, the production scheduling to feed arc furnaces in a mining-metallurgical industry. Secondly, the opportunity for disseminating and implementation of optimization concepts. Finally, this work contributes to assisting the decision-makers in solving the problems.

1.2. Objectives

1.2.1. General objective

This work aims to develop a decision support system using heuristic and mathematical programming techniques to support decision-making in the production schedule to feed arc furnaces seeking to minimize production time and respect the required quality specifications for the products generated.

1.2.2. Specific objectives

1. Characterize the production schedule to feed arc furnaces in a mining-metallurgical industry;
2. Apply a mathematical programming formulation to obtain the optimal blending of available raw materials;
3. Develop and implement a matheuristic algorithm for the scheduling problem;
4. Test and validate the proposed algorithms with real instances of the problem.

1.3. Methodology

The steps for achieving the proposed objectives are presented below:

1. **Theoretical basis:** Study of the essential concepts for understanding the addressed problem and its solution techniques;
2. **Literature review:** Analysis of works regarding the optimization of arc furnaces, production scheduling in industries, and blending problems;
3. **Problem characterization:** Characterization of the scheduling and blending problem under study and description of the production process;
4. **Solution proposal:** Presentation of a goal linear programming formulation for the blending problem, as well as a matheuristic algorithm for treating the scheduling problem;
5. **Validation of the proposed solution:** Execution of computational experiments to validate the proposed method, using instances generated based on actual data;

1.4. Work structure

The remainder of this dissertation is organized as follows. Chapter 2 presents a brief introduction of the main concepts and techniques used to develop the work. A literature review is done in Chapter 3. In Chapter 4, the problem is characterized. Chapter 5 shows the formulation developed for the blending problem. Chapter 6 presents the proposed matheuristic algorithm for the production scheduling problem. Computational experiments are reported and discussed in Chapter 7. Finally, the conclusions and proposals for future work are presented in Chapter 8.

2. Theoretical Background

This chapter is organized as follows. Initially, in Section 2.1, we illustrated a scheduling problem and a dispatch rule to solve it. Section 2.2 presents the blending problem and its linear programming formulation.

2.1. Scheduling problem

In general, scheduling can be understood as a process of sequencing and allocating jobs and resources. The objectives of this problem include minimizing the makespan, or weighted sum of tardiness, or weighted sum of earliness of the jobs (PINEDO, 2012).

According to Pinedo (2012), to build a solution to a scheduling problem, dispatch rules are usually used, such as Shortest Processing Time (SPT), Earliest Due Date (EDD), Least Flexible Job First (LFT), among others. As an example of a sequencing problem, consider a set $\mathcal{M} = \{1, 2, 3\}$ of identical parallel machines $i \in \mathcal{M}$ and the set $\mathcal{N} = \{A, B, C, D\}$ of jobs $j \in \mathcal{N}$. Let p_j be the processing times of these jobs showed in Table 2.1.

Table 2.1: Job processing times on identical parallel machines.

Job	Processing time
A	2
B	8
C	5
D	9

Suppose that the sequence of jobs was constructed by allocating the shortest duration job to each machine, as shown in the Gantt chart in Figure 2.1. The schedule of these jobs results in a makespan of 11 units.

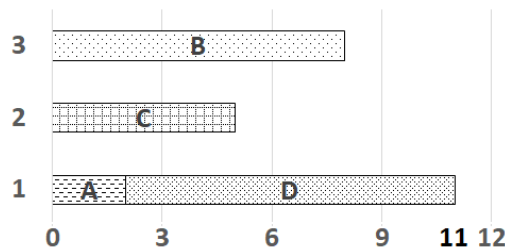


Figure 2.1: Scheduling example.

Now, we describe the dispatch rule named Adaptive Shortest Processing Time (ASPT) (HADDAD *et al.*, 2011). It is an adaptation of the Shortest Processing Time (SPT) dispatch rule (BAKER, 1974), which runs jobs in ascending processing times. In this way, the first job to be executed is the one with the shortest processing time.

To show how it works, consider the set \mathcal{M} of unrelated parallel machines and the set \mathcal{N} of jobs with their processing times p_{ij} presented in Table 2.2. According to the ASPT rule, for each pair (i, j) , where $i \in \mathcal{M}$ and $j \in \mathcal{N}$, the cost value g_{ij} of processing the job j at the end of the machine i is evaluated. Then, the pair with the lowest cost is chosen, and we allocate the job j in the last position of the machine i . This procedure is repeated until all jobs are allocated.

Table 2.2: Processing time of jobs on unrelated parallel machines.

Jobs	Machines		
	1	2	3
A	2	4	3
B	8	6	10
C	5	4	7
D	9	12	8

Algorithm 1, adapted from Cota (2014), presents the pseudo-code of the ASTP rule. After initializing solution s in line 1, a loop is executed in lines 2 to 7. In this loop, we choose the job j to be executed in the machine i with the smallest cost value g_{ij} . Then, in line 5, this job is included in the partial solution s and removed in line 6 from the set \mathcal{N} . We apply this procedure while there are jobs to execute.

Algorithm 1: ASPT

Data: Set \mathcal{M} of machines, Set \mathcal{N} of jobs
Result: Solution s

```

1  $s \leftarrow \emptyset$ ;
2 while ( $|\mathcal{N}| > 0$ ) do
3    $g_{\min} \leftarrow \min\{g_{ij} \mid i \in \mathcal{M}, j \in \mathcal{N}\}$ ;
4   Select the job  $j$  and the machine  $i$  associated with the  $g_{\min}$ ;
5    $s \leftarrow s \cup \{j\}$ ;
6    $\mathcal{N} \leftarrow \mathcal{N} \setminus \{j\}$ ;
7 end
8 return  $s$ ;
```

2.2. Blending problem

According to Arenales *et al.* (2007), the blending problem consists of combining materials that present specific quality parameters to generate products with desirable characteristics. This type of problem is found in several areas such as metal alloys, filter composition, fertilizer production, among others. Typically, the blending problem is a sub-problem of a more general problem, such as production planning.

Let the input parameters and decision variables defined below. The blending problem can be modeled through Equations (2.1)–(2.8):

a) Input parameters:

- M : Set of materials, $M = \{1, \dots, |M|\}$;
- S : Set of material quality parameters, $S = \{1, \dots, |S|\}$;
- t_{ab} : Grade of the parameter $b \in S$ of material $a \in M$, in %;
- c_a : Cost of one tonne of material $a \in M$;
- Q_{des} : Total amount desired for the blending, in tonnes;
- tl_b : Lower bound for the parameter $b \in S$ in the blending, in %;
- tu_b : Upper bound for the parameter $b \in S$ in the blending, in %;
- Q_a : Quantity available for material $a \in M$, in tonnes;

b) Decision variables:

- x_a : Quantity to be used of the material $a \in M$ in the blend, in tonnes;

c) Objective function:

$$\min \sum_{a \in M} c_a \times x_a \quad (2.1)$$

d) Constraints:

1. The sum of the quantities of materials used must result in the desired quantity for the blend:

$$\sum_{a \in M} x_a = Q_{des} \quad (2.2)$$

2. The value of the quality parameter $b \in S$ cannot be less than its predefined minimum value:

$$\frac{\sum_{a \in M} t_{ab} \times x_a}{\sum_{a \in M} x_a} \geq tl_b \quad \forall b \in S \quad (2.3)$$

Linearizing Equation (2.3):

$$\sum_{a \in M} (t_{ab} - tl_b) \times x_a \geq 0 \quad \forall b \in S \quad (2.4)$$

3. The value of the quality parameter $b \in S$ cannot be greater than its predefined maximum value:

$$\frac{\sum_{a \in M} t_{ab} \times x_a}{\sum_{a \in M} x_a} \leq tu_b \quad \forall b \in S \quad (2.5)$$

Linearizing Equation (2.5):

$$\sum_{a \in M} (t_{ab} - tu_b) \times x_a \leq 0 \quad \forall b \in S \quad (2.6)$$

4. The amount of material used is limited by its available quantity:

$$x_a \leq Q_a \quad \forall a \in M \quad (2.7)$$

5. The amount of material used cannot be negative:

$$x_a \geq 0 \quad \forall a \in M \quad (2.8)$$

3. Literature Review

A literature review is performed in this chapter. In Section 3.1, works treating optimization in arc furnaces are presented. In Section 3.2, scheduling problems in mining and metallurgical industries are treated. Finally, Section 3.3 deals with blending problems in these industries.

3.1. Arc furnaces

Several works related to arc furnaces deal with reducing energy consumption since this consumption represents a high cost.

Hernández *et al.* (2020) seek an optimal melting profile to minimize energy losses during production batches using mathematical models of an arc furnace. The models are solved by differential equations, while a control vector parametrization technique obtains the optimum values for the setpoints.

Saboochi *et al.* (2019) propose a framework to define various settings for the control of an arc furnace, such as the employed power, carbon injection, among others. Based on models of an arc furnace and their constraints, they try to increase their efficiency by maximizing the energy transferred to the scrap during refining and, consequently, reduce operating costs. The authors combine optimization algorithms such as genetic algorithms (GA) (HOLLAND and HOLLAND, 1975) and sequential quadratic programming (SQP) (BOGGS and TOLLE, 1995) to solve the proposed model. The results show a reduction in energy consumption, in contrast to an increase in additives, such as oxygen and carbon.

Lin *et al.* (2012) propose a Multi-objective Particle Swarm Optimization (MOPSO) (MOORE *et al.*, 2000) algorithm to reduce electricity consumption, refining time, electrode consumption, and extend the life of the interior furnace. As an output of the algorithm, the developed power supply model achieves the objectives established in the simulations performed.

3.2. Scheduling in mining and metallurgical industries

There are several approaches in the literature to solve scheduling problems. The most common is applying heuristic methods, given the complexity of the problem (PINEDO, 2012).

Gomes *et al.* (2021) develop a matheuristic to schedule the heat treatment line of a multinational steel company, seeking to minimize the total tardiness and energy costs. The initial solutions for each objective are generated by a Mixed-Integer Linear Programming (MILP) formulation. The Multi-objective General Variable Neighborhood Search (MOVNS) (GEIGER, 2008) metaheuristic is used to explore the solution space. Tests carried out with industry data showed reductions in energy consumption and tardiness.

Baykasoğlu and Ozsoydan (2018) present a study of dynamic scheduling of production

in heat treatment furnaces. Events such as machine breakdown, change in due dates, change in task priorities, and the arrival of new jobs are considered in the scheduling. They used the Greedy Randomized Adaptive Search Procedure (GRASP) (RESENDE and RIBEIRO, 2003) algorithm to generate the solution. In the pilot study carried out, the proposed solution reduced the total downtime of the furnaces and the consequent increase in production.

Araujo *et al.* (2008) treat the problem of batch sizing and scheduling production in small foundries in Brazil. The developed model considers characteristics such as a planning horizon, variation of the furnace capacity, stock generation, the penalty for delay, and changing the furnace setup. The authors developed two local search methods and used the Simulated Annealing (SA) (HENDERSON *et al.*, 2003) metaheuristic to find better solutions.

For works that address sequencing problems, we can cite in the literature (COTA *et al.*, 2014, 2019; HADDAD *et al.*, 2015; SABERI-ALIABAD *et al.*, 2020).

3.3. Blending in mining and metallurgical industries

Goal linear programming is widely used to solve the blending problem (CHANDA and DAGDELEN, 1995; COSTA *et al.*, 2005; MORAES *et al.*, 2006). This optimization technique is used in these works to minimize the quality deviations of the generated products. Other approaches to the problem are also used, such as fuzzy logic in Xu *et al.* (2008) and stochastic optimization in Lingshuang *et al.* (2013).

Yuan *et al.* (2020) propose an optimization model for blending coke to be used in blast furnaces. The goal is to determine the best blend for coke at the lowest cost. Initially, the authors seek to estimate the quality of the coke using Gaussian functions and the Extreme Gradient Boosting Algorithm (XGBoost) (CHEN and GUESTRIN, 2016) to select the most relevant characteristics. Then they apply the Support Vector Regression (SVR) (VAPNIK, 1995) algorithm to forecast the products to be generated. At least, the blending optimization model is solved using a modified version of the Particle Swarm Optimization (PSO) (KENNEDY and EBERHART, 1995) algorithm.

A similar approach to the preparation of the sinter blend is covered in Zhang *et al.* (2019). Seeking to maximize profit in the blending of sinter, the authors use the SVR algorithm to estimate the sinter classification based on the properties of the raw materials and, thus, price the generated blend. Finally, they use the Non-dominated Sorting Genetic Algorithm II (NSGA-II) (DEB *et al.*, 2000) algorithm to solve the blending problem. The authors report a reduction in the cost of the sinter and an increase in production profit.

Huang *et al.* (2019) aim to reduce energy consumption and production costs with an optimization model for the sinter blend. Initially, the quantities of raw materials that make up the blending are defined considering, in addition to the different qualities, the granulation, and mineralization properties. An initial solution for the blend is generated through linear programming. This solution is used as input to a GA, which generates a group of blending schemes.

Then, the chemical properties of the raw materials of these blending schemes are used as inputs of the Least Squares Support Vector Machine (LS-SVM) (SUYKENS and VANDEWALLE, 1999) algorithm for the prediction of energy consumption and productivity in the sintering process. Finally, the blends and their respective blending cost, energy consumption, and productivity rates are evaluated. The blending scheme with the highest economic and technical value is selected. The authors report a significant reduction in carbon emissions for the considered process.

4. Problem Statement

The problem of sequencing products to feed the arc furnace under study has the following characteristics:

1. There is a set M of ores of different qualities to be used as raw material, $M = \{1, \dots, |M|\}$;
2. There is a set SM of silos to store the raw material, $SM = \{1, \dots, |SM|\}$;
3. There is a set PR of products to be generated from the blend of raw materials, $PR = \{1, \dots, |PR|\}$;
4. There is a set S of quality parameters for raw materials and products, $S = \{1, \dots, |S|\}$;
5. There is a production planning horizon H , in minutes;
6. There is a set SF of screw feeders, $SF = \{1, \dots, |SF|\}$;
7. Each screw feeder $sf \in SF$ belongs to a raw material silo $a \in SM$;
8. There is a set CB of conveyor belts, $CB = \{1, \dots, |CB|\}$;
9. Each product $j \in PR$ is associated with a processing time p_j and a due date d_j ;
10. Each product $j \in PR$ is associated with a type of raw material;
11. Every silo $a \in SM$ stores a maximum of Qu tonnes of raw material;
12. Every silo $a \in SM$ must store at least Ql tonnes of raw material;
13. The material contained in each raw material silo $a \in SM$ is associated with a type of raw material;
14. The grade of the parameter $b \in S$ in the raw material silo $a \in SM$ is given by t_{ab} , in %;
15. The recommended grade of the parameter $b \in S$ in the product $j \in PR$ is given by tr_{jb} , in %;
16. For each product $j \in PR$, there is a lower and upper bound for the control parameter $b \in S$, given by tl_{jb} and tu_{jb} , in %, respectively, to be met;
17. When a raw material silo is being fed, it cannot be used in the productive process.

The goal is to generate a production sequence that meets the required quality specifications and minimizes the weighted sum of the *makespan* and the total tardiness, given by:

$$\min \alpha C_{\max} + (1 - \alpha) \sum_{j \in PR} T_j \quad (4.1)$$

where:

1. $\alpha \in [0.1]$ is a parameter that reflects the importance of each parcel of the objective function;
2. C_j is the completion time of the product j , in minutes;
3. $C_{\max} = \max\{C_j, j \in PR\}$, in minutes;
4. $T_j = \max\{C_j - d_j, 0\}$ is the tardiness of the product j , in minutes.

The following images describe the production process. Figure 4.1 illustrates raw material silos and the tripper car to distribute the material into the silos. As Wills and Finch (2015), a tripper car is a machine capable of moving on rails and positioning itself above the silos, allowing its load to be delivered to one or more silos. In the analyzed process, one silo is fed at a time. Each raw material silo receives a type of ore with a given chemical composition, determined by laboratory analysis. The material that feeds each of the raw material silos comes from a previous process step, not covered in this work.

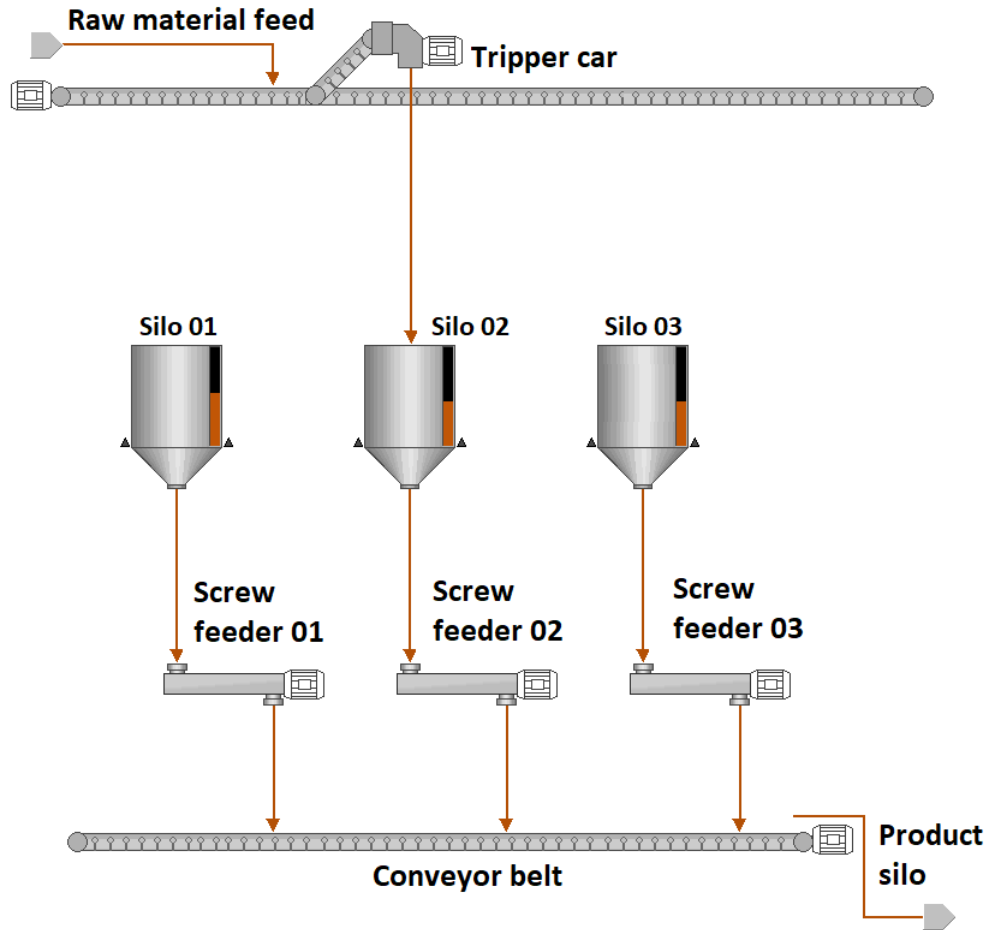


Figure 4.1: Example of setting up silos for storing blends and tripper car.

Below each silo of raw material, there is a screw feeder, equipment responsible for removing the desired amount of material to compose the product to be generated. This removed

material is then deposited on conveyor belts, which send it to the product storage silo, represented in Figure 4.2. Once the product is generated, it is then consumed in the next stage of the production process, that is, in the feeding of the arc furnace.

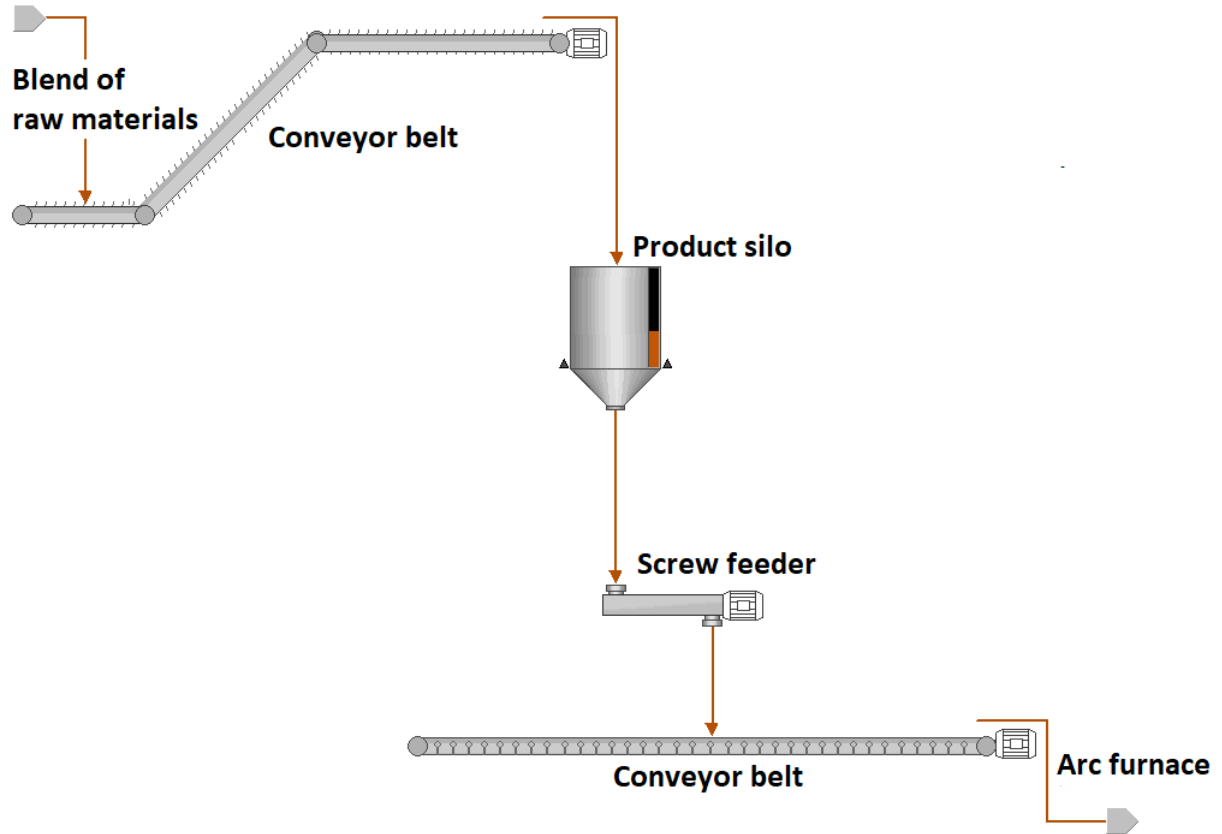


Figure 4.2: Example of setting up silos for storing blends.

To illustrate a solution to this problem, let Table 4.1 be an example of a production planning to be executed. Table 4.1 reports the product characteristics in this sequence: product ID, mass, type of material, the due date, and its respective desired chemical composition.

Table 4.1: Production planning to be executed.

Products	Mass (tonnes)	Material	Due date (min)	Parameters (%)				
				A	B	C	D	E
1	18	0	480	21.65	0.87	0.66	68.59	4.32
2	18	3	960	24.11	2.26	1.23	69.17	4.16
3	18	1	960	24.39	1.32	0.59	68.53	5.38
4	21	2	480	22.75	1.92	1.17	67.49	4.29

Table 4.2 presents the values of mass, type of material, and the grade of each control parameter of the ores contained in the silos of raw materials in the initial instant.

From Tables 4.1 and 4.2, it is possible to calculate the composition of the blending

Table 4.2: Characteristics of raw material silos.

Silo	Mass (tonnes)	Material	Parameters (%)				
			A	B	C	D	E
1	67.00	0	24.23	2.48	0.69	68.58	4.08
2	67.51	2	20.66	2.48	1.12	67.69	5.38
3	70.18	3	24.02	0.85	1.00	68.22	5.33
4	159.65	3	24.93	2.24	1.15	69.47	4.28
5	61.23	1	21.37	1.59	0.89	68.26	5.44
6	171.16	0	26.57	0.74	1.23	69.79	5.89
7	253.44	2	24.35	0.50	1.04	68.11	5.12

required to generate each product. The raw materials used in this blend must be those available in silos that have the same type of raw material required by the product.

When defining the amount of material to be removed from each raw material silo, the screw feeders located just below these are activated, except for those whose amount of material is null. Then, the material is transferred by conveyor belts and sent to the product silo.

5. The Mathematical Formulation of the Blending Problem

The blending problem can be approached through a goal linear programming formulation, associating a deviation variable for each constraint that cannot be obeyed. These deviation variables measure distances from target values to be achieved and represent goals to be reached. So, the objective function of this formulation is formed by the weighted sum of these goals and should be minimized. We adapt the model by Moraes *et al.* (2006) to solve the blending problem. As the model is applied to generate a single product, for simplicity, we have eliminated the index j in the input data defined in items 15, and 16 of Section 4.

Input data:

1. SM : Set of raw material storage silos;
2. S : Set of quality parameters for raw materials and products;
3. PR : Set of products to be generated from the blend of raw materials;
4. t_{ab} : Grade of the parameter $b \in S$ of the raw material stored in the silo $a \in SM$;
5. tr_b : Target for the parameter $b \in S$ in the blending;
6. tl_b : Lower bound for the parameter $b \in S$ in the blending;
7. tu_b : Upper bound for the parameter $b \in S$ in the blending;
8. Q_a : Mass available in the silo $a \in SM$;
9. ws_a : Weight for the use of the silo $a \in SM$;
10. Q_{des} : Desired mass for the product;
11. wm_b : Weight for the deviations from the grade target of the parameter $b \in S$;
12. $type_a$: Binary parameter that assumes value 1 if the raw material silo $a \in SM$ has the same type of material desired for the product j to be blended and 0, otherwise;

Decision variables:

1. x_a : Quantity of mass to be taken from the raw material of the silo $a \in SM$;
2. $dnSiloSM_a$: Amount of raw material remaining in the silo $a \in SM$, used for blending, in tonnes;
3. dnm_b : Negative deviation from the grade target for the parameter $b \in S$, in tonnes;
4. dpm_b : Positive deviation from the grade target for the parameter $b \in S$, in tonnes;

The blending problem can be formulated through Equations (5.1)–(5.11).

$$\min \sum_{b \in S} w m_b \times (d p m_b + d n m_b) + \sum_{a \in S M} w s_a \times d n S i l o S M_a \quad (5.1)$$

$$\sum_{a \in S M} x_a = Q d e s \quad (5.2)$$

$$x_a \leq Q_a \quad \forall a \in S M \quad (5.3)$$

$$\sum_{a \in S M} (t_{ab} - t l_b) \times x_a \geq 0 \quad \forall b \in S \quad (5.4)$$

$$\sum_{a \in S M} (t_{ab} - t u_b) \times x_a \leq 0 \quad \forall b \in S \quad (5.5)$$

$$\sum_{a \in S M} (t_{ab} - t r_b) \times x_a + d n m_b - d p m_b = 0 \quad \forall b \in S \quad (5.6)$$

$$x_a + d n S i l o S M_a = Q_a \times t y p e_a \quad \forall a \in S M \quad (5.7)$$

$$x_a \geq 0 \quad \forall a \in S M \quad (5.8)$$

$$d n S i l o S M_a \geq 0 \quad \forall a \in S M \quad (5.9)$$

$$d n m_b \geq 0 \quad \forall b \in S \quad (5.10)$$

$$d p m_b \geq 0 \quad \forall b \in S \quad (5.11)$$

The objective function described by Equation (5.1) seeks to minimize the deviations from the grade target for all parameters and the amount of raw material remaining in the silo used for blending. The constraints applied to the model are presented by Equations (5.2) to (5.7). Constraints (5.2) determine that $Qdes$ tonnes of product will be produced. Constraints (5.3) ensure that the mass to be removed from each raw material silo cannot be greater than its available mass. Constraints (5.4) and (5.5) ensure compliance with the lower and upper specification limits, while constraints (5.6) seek to meet the grade target. Constraints (5.7) indicate that all material from a raw material silo should be removed whenever possible. Equations (5.8–5.11) establish the domain of the decision variables.

When solving the blending problem, the mathematical model returns that there is no feasible solution or, if there is, it returns a solution that satisfies the lower and upper bounds stated for the product, minimizing the weighted objective function given by Equation (5.1).

6. Proposed Algorithm

A matheuristic algorithm for decision support, named DSS_ASPT, is proposed to schedule the desired products. It combines the Adaptive Shortest Processing Time (ASPT) (BAKER, 1974) heuristic procedure to generate the production sequence and a goal linear programming formulation to solve the blending problem. Its pseudo-code is presented by Algorithm 2.

As input parameters, Algorithm 2 receives the set SM of raw material silos, the set PR of products to be generated, and the production planning horizon H . Besides, the parameter α , the minimum (Ql) and maximum (Qu) mass values for a raw material silo, and a list containing the minimum (tl_b) and maximum (tu_b) values of grades for each parameter b of a given type of material are algorithm inputs. Finally, the algorithm's outputs are the solution s containing the production sequence and its value fo according to Equation (4.1).

First, some variables, sets, and counters used in the algorithm are initialized, such as the makespan C_{\max} , the total tardiness T , the instant h of the planning horizon, and others. In line 3, the list with the limit values of the grades is obtained. In line 4, it is determined how many days make up the production planning horizon H . So a loop is initialized, and in line 6, it is determined which products $p \in PR$ have a due date for the day analyzed, generating the set PR_{Day} . These products are then removed from the set PR so that they are not analyzed in the next iteration.

In line 8, the rejected products from the set $PR_{Rejected}$ are handled by the rejected products treatment (RPT) function. Nothing will be processed in the first iteration of the loop, as there are still no rejected products. Algorithm 3 describes the pseudo-code of the RPT function (see Section 6.1.1).

Once processed the rejected products, line 9 checks which products belonging to the set PR_{Day} are candidates to be produced. Then, it is solved the blending problem for each product in the set PR_{Day} . The optimizer returns the mass values to be extracted from the raw material silos, indicating whether the product is a candidate to be generated or not. If the optimizer returns null values for the mass to be removed, the product is considered rejected. If the product is rejected, its priority is increased in one unit. The rejected products are then added to the set $PR_{Rejected}$ to be treated in the next iteration.

Then, we start a loop that operates as long as there is a product $p \in PR_{Candidates}$. First, in line 15, we select a product with the shortest completion time to be produced during the current work shift. This product is associated with all the information necessary for its production, such as mass, recommended grades, and acceptable quality limits, i.e., lower and upper bounds for its control parameters. Once the product is selected, the raw material feed (RMF) function analyzes in line 16 if there is any raw material silo that must be fed. The RMF function is described through the pseudo-code of Algorithm 4 (see Section 6.1.2).

Algorithm 2: DSS_ASPT

Data: Set SM of raw material silos, set PR of products to be generated, planning horizon H , parameter α , minimum mass value Ql , maximum mass value Qu , lower and upper bounds for the parameters.

Result: Solution s and objective function value fo .

```
1  $C_{\max}, T, h, Day, fo \leftarrow 0$ ;  
2  $PR_{Rejected}, PR_{Day}, s \leftarrow \emptyset$ ;  
3  $GradeList = GradeBounds()$ ;  
4  $NumDays = GetNumberofDays(H)$ ;  
5 while  $Day < NumDays$  do  
6    $PR_{Day} \leftarrow FindDayProducts(PR, Day)$ ;  
7    $PR \leftarrow PR \setminus PR_{Day}$ ;  
8    $(s, PR_{Rejected}) = RPT(SM, PR_{Rejected}, h, PR, Day, Ql, Qu, GradeList)$ ;  
9    $(PR_{Candidates}, PR_{RejectedDay}) = FindPossibleProducts(PR_{Day}, SM)$ ;  
10  while  $PR_{RejectedDay} \neq \emptyset$  do  
11     $IncreasePriority(PR_{RejectedDay})$ ;  
12     $PR_{Rejected} \leftarrow PR_{Rejected} \cup PR_{RejectedDay}$ ;  
13  end  
14  while  $PR_{Candidates} \neq \emptyset$  do  
15     $(p, X) = FindBestProduct(PR_{Candidates}, SM)$ ;  
16     $RMF(SM, PR, PR_{Rejected}, Day, Ql, Qu, GradeList)$ ;  
17    if there are sufficient mass in raw material silos then  
18       $h = h + p_j$ ;  
19       $C_j = h$ ;  
20       $T = T + \max\{C_j - d_j, 0\}$ ;  
21       $UpdateMass(X, SM)$ ;  
22       $s \leftarrow s \cup \{p\}$ ;  
23       $PR_{Candidates} \leftarrow PR_{Candidates} \setminus \{p\}$ ;  
24    end  
25    else  
26       $IncreasePriority(p)$ ;  
27       $PR_{Rejected} \leftarrow PR_{Rejected} \cup \{p\}$ ;  
28       $PR_{Candidates} \leftarrow PR_{Candidates} \setminus \{p\}$ ;  
29    end  
30  end  
31   $Day++$ ;  
32 end  
33 if  $PR_{RejectedDay} \neq \emptyset$  then  
34   while  $PR_{RejectedDay} \neq \emptyset$  do  
35      $RMF(SM, PR, PR_{Rejected}, Day, Ql, Qu, GradeList)$ ;  
36      $(s, PR_{rejeitados}) = RPT(SM, PR_{Rejected}, h, PR, Day, Ql, Qu, GradeList)$ ;  
37   end  
38 end  
39  $C_{\max} = h$ ;  
40  $fo = CalculateFO(C_{\max}, T, \alpha)$ ;  
41 return  $s, fo$ ;
```

Next, we check whether the raw material silos at that time analyzed contain sufficient mass to generate the product p . The product is rejected in the lack of mass, and its priority is increased by one unit for each rejection. If the product is rejected, it is included in the set $PR_{Rejected}$ and is removed from the set $PR_{Candidates}$ in lines 26 to 28.

If there are sufficient raw materials in silos, the time analyzed h is updated by increasing it with the processing time of the product generated. The processing time p_j is determined by the longest time spent by the screw feeders for extracting the required amount of mass from the raw material silos. As the feeders are activated simultaneously, the one that consumes the most time to finish its activity will be the one that will determine the total duration of the product processing. The makespan C_{\max} , the total tardiness T , and the mass present in each silo of raw material are updated using the *UpdateMass(.)* function. Then, in line 22, the partial solution s is updated with the product p and then excluded from the set $PR_{Candidates}$. The loop ends when the set $PR_{Candidates}$ is empty.

Finally, we increase the variable *Day* in line 31, and the loop continues until all the days of the planning horizon are analyzed.

After evaluating the products for each day of the planning horizon, there may still be rejected products. Therefore, they are treated in the loop initialized in line 34 until all products have been generated, ending the production schedule.

6.1. Complementary functions

This section presents the functions that contribute to the execution of the DSS_ASPT algorithm.

6.1.1. RPT function

This function does the treatment of the rejected products in the DSS_ASPT algorithm. It receives as input parameters the set SM of raw material silos, the set $PR_{Rejected}$ of rejected products, the time h , the mass limit values for a silo of raw material (Qu, Ql), and the lower and upper grade bounds (tl_{jb}, tu_{jb}) for each parameter b in the product j . The output is the solution s updated and the set of rejected products $PR_{Rejected}$.

Algorithm 3 describes its pseudo-code. First, we check which product can be generated among the rejected products. Then, we initialize a loop to investigate the candidate products. In line 3, the product of the set $PR'_{Candidates}$ with the highest priority and the shortest duration to generate it is selected. Then, we check by the RMF function if there is an empty raw material silo that must be fed.

Suppose there is a sufficient mass of raw material. In that case, we generate the selected product, update the makespan C_{\max} , the total tardiness T of the solution, and the mass present in each raw material silo. We update the partial solution s with the product p in line 10 and exclude

Algorithm 3: RPT

Data: Set SM of raw materials silos, set $PR_{Rejected}$ of rejected products, time h , set PR of products, analyzed day Day , minimum mass value Ql , maximum mass value Qu , lower and upper bounds for the parameters.

Result: Solution s and set $PR_{Rejected}$.

```
1 ( $PR'_{Candidates}, PR'_{Rejected}$ ) =  $FindPossibleProducts(PR_{Rejected}, SM)$ ;
2 while  $PR'_{Candidates} \neq \emptyset$  do
3   ( $p', X$ ) =  $FindPriorityProducts(PR'_{Candidates}, SM)$ ;
4    $RMF(SM, PR, PR_{Rejected}, Day, Ql, Qu, GradeList)$ ;
5   if there are sufficient mass in raw material silos then
6      $h = h + p'_j$ ;
7      $C'_j = h$ ;
8      $T = T + \max\{C'_j - d'_j, 0\}$ ;
9      $UpdateMass(X, SM)$ ;
10     $s \leftarrow s \cup \{p'\}$ ;
11     $PR'_{Candidates} \leftarrow PR'_{Candidates} \setminus \{p'\}$ ;
12  end
13  else
14     $IncreasePriority(PR_{Rejected}[p'])$ ;
15     $PR'_{Candidates} \leftarrow PR'_{Candidates} \setminus \{p'\}$ ;
16  end
17 end
18 while  $PR'_{Rejected} \neq \emptyset$  do
19    $IncreasePriority(PR_{Rejected})$ ;
20 end
21 return  $s, PR_{Rejected}$ ;
```

it from the set $PR'_{Candidates}$. If there is not enough mass, we reject the product and increase its priority in line 14. Products that were not considered candidates have their priority increased in line 19.

6.1.2. RMF function

This function executes the feed of a raw material silo identified as empty. Algorithm 4 describes its pseudo-code.

Initially, we initialize the variables. In line 2, we check if there is an empty raw material silo. A silo is considered empty if its mass value is less than Ql . If it is empty, the variable $EmptySilo$ assumes the ID of the empty silo. Otherwise, it receives the value 0.

Lines 3–5 identify the most demanded raw material for both the products of the following day of the planning horizon and rejected products. Knowing the raw material to be fed, the empty silo is fed until the mass value reaches the value Qu (line 6).

When defining the raw material silo to be fed, the RMF function also determines the position of the tripper car along the production planning horizon.

Algorithm 4: RMF

Data: Set SM of raw materials silos, set PR of products, set $PR_{Rejected}$ of rejected products, analyzed day Day , minimum mass value Ql , maximum mass value Qu , list of grades for each type of material.

Result: Set SM .

```
1  $EmptySilo, MaterialType, MPR_{NextDay}, MPR_{Rejected} \leftarrow 0$ ;  
2  $EmptySilo = FindEmptySilo(SM, Ql)$ ;  
3  $MPR_{NextDay} = FindNextDayMaterial(PR, Day)$ ;  
4  $MPR_{Rejected} = FindProductRejectedMaterial(PR_{rejeitados})$ ;  
5  $MaterialType = MostRequestedType(MPR_{NextDay}, MPR_{Rejected}, PR, PR_{Rejected})$ ;  
6  $SM = FeedSilo(EmptySilo, MaterialType, Qu, GradeList)$ ;  
7 return  $SM$ ;
```

7. Computational Experiments

The DSS_ASPT algorithm was implemented using the C# programming language on the .NET Framework 4.7.2 platform in the Microsoft Visual Studio Community 2017 development environment. We execute the tests on a computer with an Intel Core i7-8550U @ 1.80 GHz \times 8 processor, with 16 GB of RAM and Windows 10 64-bit operating system.

To test the algorithm, we generate seven instances based on actual production data from a mining-metallurgical industry. Each instance has the number of raw material silos, the number of products to be generated, the planning horizon, and the due date of each product per work shift. We generate these instances according to the schema below:

1. The flow rate of the screw feeders was set at 30 tonnes/h, and the feed flow of the tripper car at 28 tonnes/h;
2. The mass of the product $j \in PR$ is chosen randomly in the range $[0.7 \times 24, 24]$, where the value 24 represents the maximum mass of the product to be generated in tonnes;
3. The maximum capacity of each raw material silo is $Qu = 285$ tonnes;
4. The raw material type for generate the product j is chosen randomly in the range $[0, 3]$ to represent each of the four products generated by the company under study;
5. The grades t_{ab} of the mass stored in the raw material silo $a \in SM$ are chosen randomly in the range $[tl_{ab}, tu_{ab}]$, according to the type of raw material stored in silo a ;
6. The recommended grades tr_{jb} are chosen randomly in the range $[tl_{jb}, tu_{jb}]$ according to the type of the product j ;
7. The due date d_j of the product $j \in PR$ is chosen according to the number of products per work shift, that is, 9 or 10;
8. The mass of material stored in the raw material silo $a \in SM$ is chosen randomly in the interval $[0.7 \times 24, Qu]$;
9. The raw material type stored in silo a is chosen randomly in the range $[0, 3]$.

Table 7.1 shows the characteristics of the instances generated. It has the following organization: *i*) the first column: index of the instance; *ii*) the second column: the total of products to be generated; *iii*) the third column: the number of products with due date per shift work; *iv*) the fourth column: number of raw material silos available; and *v*) the fifth and sixth: the production planning horizon. The number of silos represents exactly the structure present in the studied industry. We will not disclose the quality parameters and raw materials due to industrial secrecy.

Table 7.1: Characteristics of the instances.

# Instance	# Products		# Raw material silos	Planning horizon	
	Per work shift	Total		Minutes	Days
1	9	30	14	1,440	1
2	9	81	14	4,320	3
3	10	90	14	4,320	3
4	9	189	14	10,080	7
5	10	210	14	10,080	7
6	9	270	14	14,400	10
7	10	300	14	14,400	10

We modeled the blending problem through the goal linear programming formulation described in Section 5 and used the LINGO modeler and optimizer software from Lindo Systems Inc., version 10, to solve it. In the developed code, the blending input data are passed to the optimizer through a DLL. The optimizer, in turn, returns the mass to be extracted from each raw material silo. The weights of quality parameters in the model were classified as *Very Important*, *Critical*, and *Very Critical*, with weights set to 5, 10, and 100, respectively, according to the classification of Moraes *et al.* (2006). Table 7.2 shows the weights used for each quality parameter considered in the model. The weights ws_a were set to a high value (bigM) for all silos.

Table 7.2: Weights for the quality parameters.

	Quality parameters				
	A	B	C	D	E
Weight	5	5	10	100	5

We execute the DSS_ASPT algorithm considering equal importance to the makespan and total tardiness objectives, that is, the parameter α of Equation (4.1) was set to 0.5.

Figure 7.1 shows the graphical interface of the decision support system. The user can define the number of raw material silos, the number of products to be generated, the planning horizon, the process flow rates, the maximum mass, and other parameters. Then, when activating the Run command, the system suggests a product schedule.

Table 7.3 presents the algorithm results. The first three columns report the instance identifier, the makespan in minutes, and the total tardiness in minutes, respectively. The fourth column shows the longest tardiness, in minutes, among the sequenced products. The value of the objective function generated by the solution is presented, in minutes, in the fifth column. Its sixth column reports the total execution time in seconds. In the last five columns, we report the Average Relative Percentage Deviation (ARPD_b) for the parameter b in each instance, calculated according to Equation (7.1):

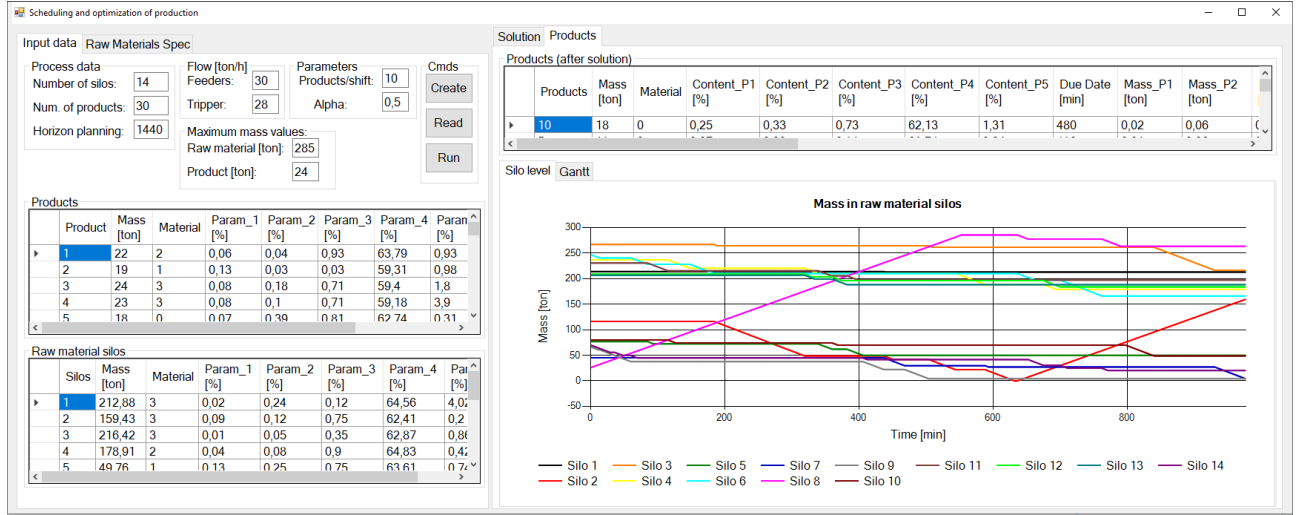


Figure 7.1: User interface for the decision support system.

$$ARPD_b = \frac{\sum_{j \in PR} |te_{jb} - tr_{jb}| / tr_{jb}}{|PR|} \quad (7.1)$$

where te_{jb} is the the value encountered for the parameter b in the product j .

Table 7.3: Results of the proposed algorithm per instance.

Instance	Makespan	Total	Maximum	Objective	Runtime	Average Relative Percentage Deviation				
	(min)	tardiness (min)	tardiness (min)	function (min)	(seg)	A	B	C	D	E
1	660.24	0.00	0.00	330.12	5	0.10	0.16	0.03	0.00	0.08
2	2,114.56	0.00	0.00	1,057.28	16	0.18	0.12	0.04	0.01	0.13
3	2,853.49	0.00	0.00	1,426.75	21	0.14	0.14	0.03	0.01	0.06
4	8,189.96	2,379.56	834.87	5,284.76	67	0.18	0.11	0.03	0.01	0.09
5	10,191.08	62,761.94	2,564.33	36,476.51	157	0.14	0.21	0.04	0.01	0.09
6	12,526.98	76,006.57	3,537.69	44,226.78	215	0.15	0.13	0.05	0.01	0.06
7	14,483.00	122,737.01	4,178.04	68,610.01	267	0.15	0.15	0.07	0.01	0.04

Table 7.3 shows that algorithm DSS_ASPT sequences all products. However, the planning horizon is extrapolated in some instances. This extrapolation occurs in two instances with more products per work shift. Regarding the total tardiness, although this value was high in some instances, such as in instance 7, the maximum tardiness was less than three days. These facts may indicate an oversizing of the production target; fewer products should be produced per work shift in these instances. The deviations from the quality parameters varied according to the priority assigned to each one. Thus, parameters with the highest priority had the smallest deviations. In this case, control parameter D, which has the highest priority, had a maximum deviation of 1%. Parameter C, which has the second-highest priority, had a maximum deviation of 7%. The other parameters had a higher deviation, of up to 21%, to reach the target values. However, it is worth noting that the solution proposed by the algorithm satisfies the lower and upper bounds of the control parameters. We also highlight that the proposed algorithm consu-

mes a low runtime, with the highest equal to 267 seconds.

Figures 7.2–7.5 shows the behavior of the mass of the materials present in the raw material silos throughout the sequencing. It is possible to observe the feeding behavior of the empty silos. It is also possible to observe the instants in which only rejected products are treated. In such instants, we observe that few products are generated due to the unavailability of raw materials. Yet, at the same time, the empty silo is fed with the most requested material type to meet the demand for rejected products.

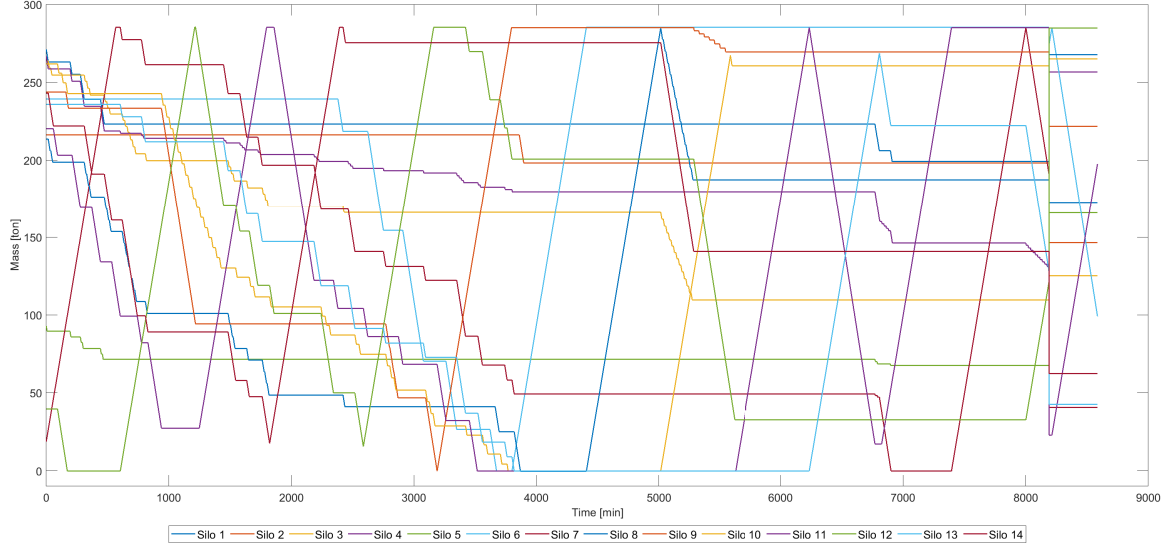


Figure 7.2: Behavior of raw material silos for instance 4.

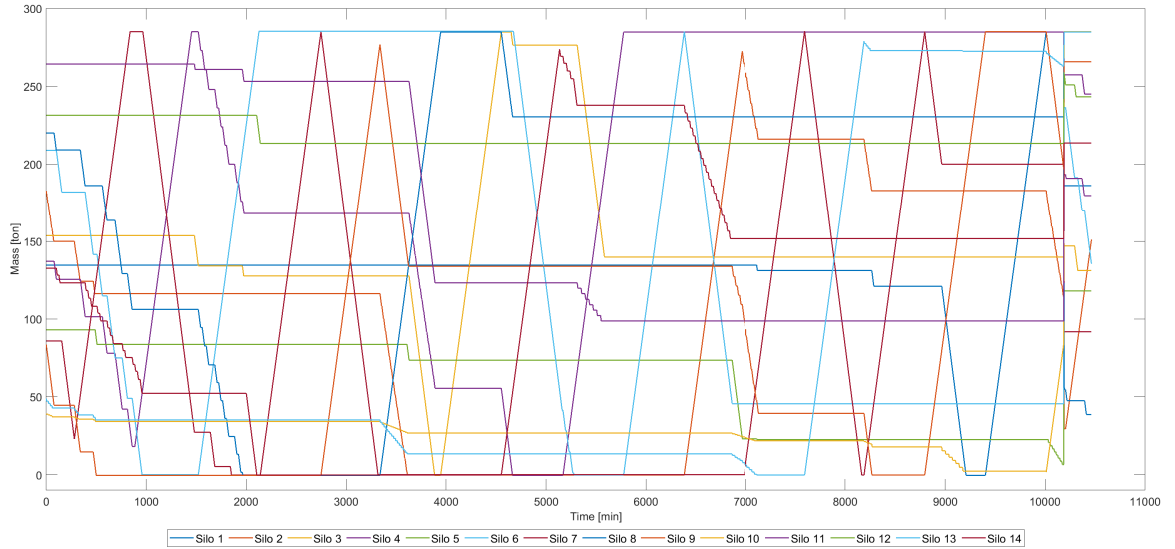


Figure 7.3: Behavior of raw material silos for instance 5.

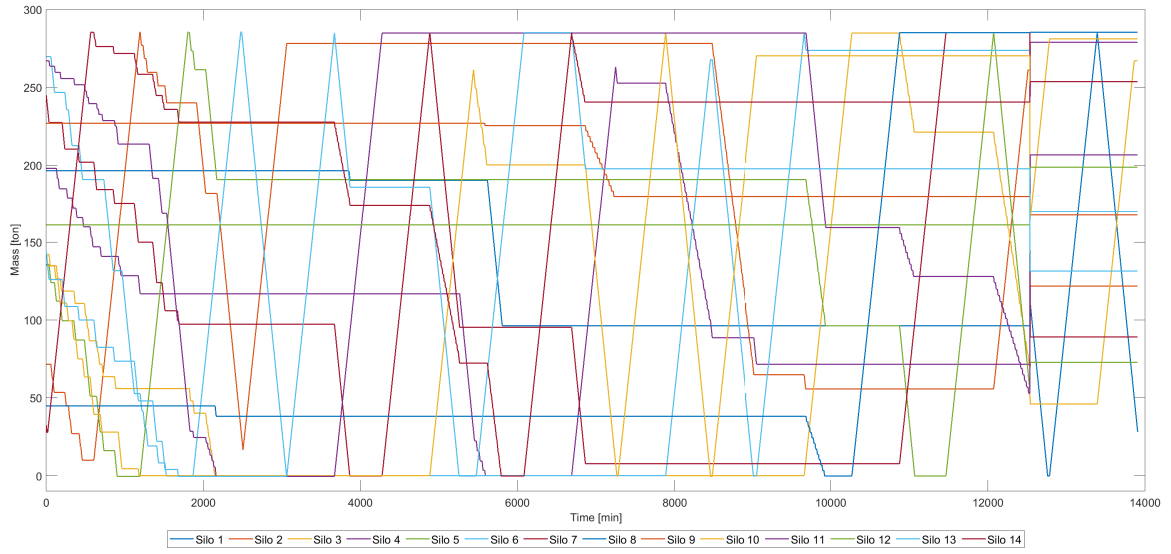


Figure 7.4: Behavior of raw material silos for instance 6.

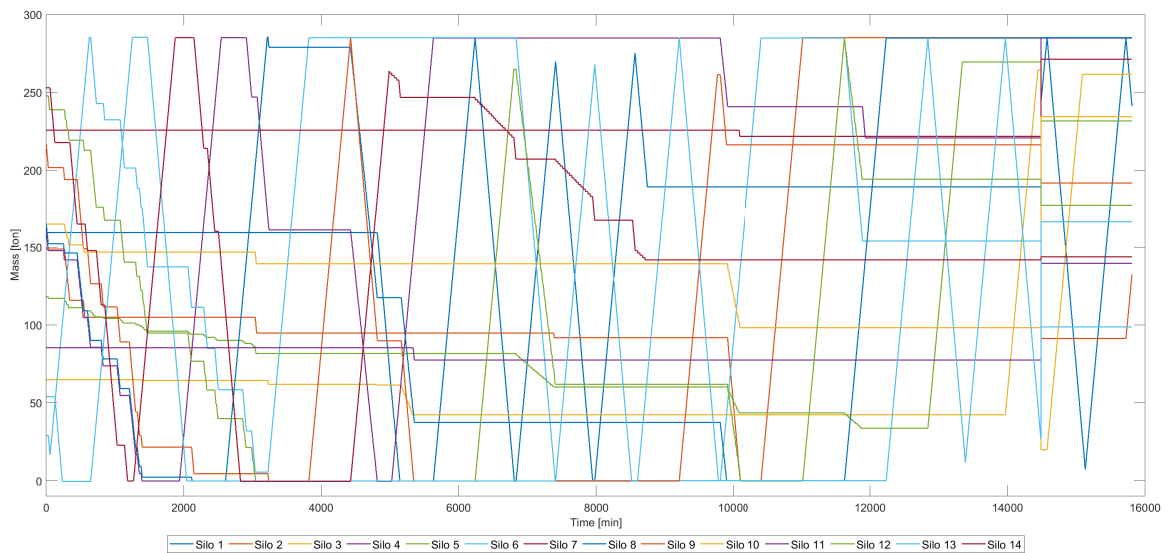


Figure 7.5: Behavior of raw material silos for instance 7.

Figure 7.6 illustrates through the Gantt chart the schedule generated by the developed algorithm for the instance 1. In this figure, each colored rectangle represents a product. The products are showed in the production sequence, and the width of each rectangle indicates the processing time of the respective product. In this sequencing, the DSS_ASPT generated a solution with a makespan of 660.24 minutes, respecting the desired production planning horizon of 1440 minutes.

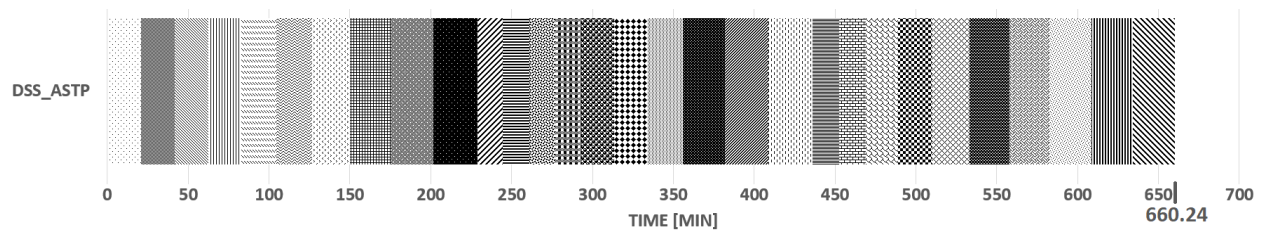


Figure 7.6: Schedule generated for the instance with 30 products.

8. Conclusions and Future Works

This work dealt with a production scheduling problem to feed an arc furnace in a mining-metallurgical industry. The objective of the schedule is to minimize the makespan and the total tardiness. To solve it, we developed a matheuristic algorithm called DSS_ASPT. First, we choose the products through a heuristic dispatch rule. According to this rule, we select the products with the shortest processing times among those whose due dates correspond to the work shift analyzed. Second, the proposed algorithm uses a goal linear programming formulation to solve the blending problem of raw materials, a sub-problem of the sequencing.

We use seven instances to test the proposed algorithm. These instances contain 30, 90, 210, and 300 products to be produced during the planning horizon in three daily work shifts. Four of them have 10 products per work shift, and three have 9 products. In these instances, the production planning horizons can be one, three, seven, and ten days.

Considering the established experimental context, the results showed that the DSS_ASPT algorithm generates all the requested products. In some instances, the makespan exceed the planning horizon, and there may be a delay in production. We can observe that the planning horizon is respected in instances with the least number of products per work shift. This result can be indicative of the ideal number of products per work shift in the industry under study. Regarding total tardiness, as the number of products to be generated grows, the delay in their production also increases. Regarding the quality parameters of the products, it is clear that the blending problem was successfully solved. The deviations from the quality parameters with the highest weights were minimal, and those from the other parameters were not high.

In addition to sequencing all required products, respecting production specifications, and reducing production delay, the developed algorithm has a low runtime. In this way, we validate its use to support decision-making in the company.

In the method proposed, a product is chosen to be inserted into the solution greedily at each step. The advantage of this type of algorithm is that a solution is found quickly and, in general, has good quality. On the other hand, the disadvantage is that greedy strategies do not always provide the best solutions to a problem. In this sense, we propose as future works to develop metaheuristic-based algorithms to generate even better solutions, such as the Iterated Local Search and Genetic Algorithm. In addition, we propose to evaluate the proposed method in instances of other similar industries and adapt it to handle other production processes that use, for example, more than one conveyor belt to transport the raw materials.

9. Publications

So far, this research has resulted in two papers: one published in the proceedings of a scientific event and another submitted to a journal. Below are their details.

1. **Title:** Sequenciamento de produtos em fornos a arco: Um estudo de caso em uma indústria mínero-metalúrgica

Authors: Rafael de Freitas Bacharel, Marcone Jamilson Freitas Souza e Luciano Perdigão Cota

Event: XXIII Congresso Brasileiro de Automática 2020

Local: Virtual

Period: November 23rd to 26th, 2020

2. **Title:** Product sequencing and blending of raw materials to feed arc furnaces: A decision support system for a mining-metallurgical industry

Authors: Rafael de Freitas Bacharel, Marcone Jamilson Freitas Souza e Luciano Perdigão Cota

Journal: Journal of Control, Automation and Electrical Systems

Observations: Accept for publishing.

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